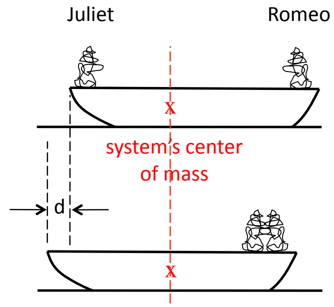
Problem 9.43

77.0 kg Romeo sits at the rear of a stationary 2.70 meter long boat serenading 55.0 kg Juliet, who is sitting in the front of the 80.0 kg boat. She gets up and walks to the rear to give Romeo a kiss (the hussy). How far does the boat move "d" as a consequence of her brazenly lustful act?

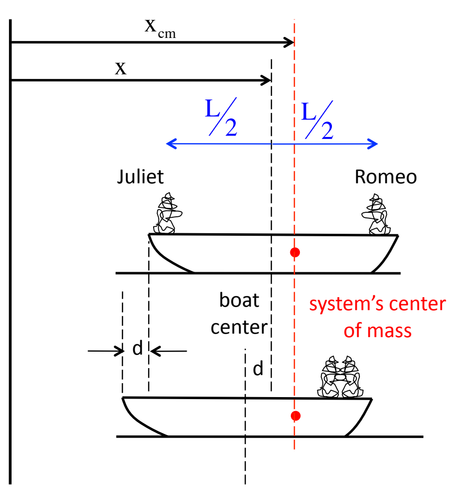


As she walks, Juliet will apply a force to the boat motivating it to move to the left. The boat will apply a N.T.L. "reaction force" to Juliet motivating her to move to the right. But though pieces of the system are moving, the *center of mass* will remain stationary because there will be no *external forces* acting on the boat/Romeo/Juliet system (we are ignoring frictional effects between the boat and the water--that's why the problem stipulated that she moved "carefully").

Using that information isn't quite enough to solve the problem. First, we need to define a coordinate axis. That is shown on the next page.

What we need to write out are two *center of mass* calculations, one for before the boat moves and one for after. As the net *center of mass* coordinate has to be the same in each case, we can equate our two equations. To that end:

Let "x" be the distance from the shore to the initial position of the boat's center. If we let the boat's length be "L," then the initial position of Juliet is "x - L/2" (look at the sketch) and the initial position of Romeo is "x + L/2." Assuming the boat's mass is all located at its geometric center (i.e., half way between the front and the back at "L/2," relative to the boat), the initial *center of mass* of the system, relative to the shore, is:



$$x_{cm} = \frac{\sum m_i x_i}{M}$$

$$\Rightarrow \quad Mx_{cm} = \sum m_i x_i$$

$$\Rightarrow (m_J + m_{boat} + m_R)x_{cm} = m_J\left(x - L/2\right) + m_{boat}(x) + m_R\left(x + L/2\right)$$

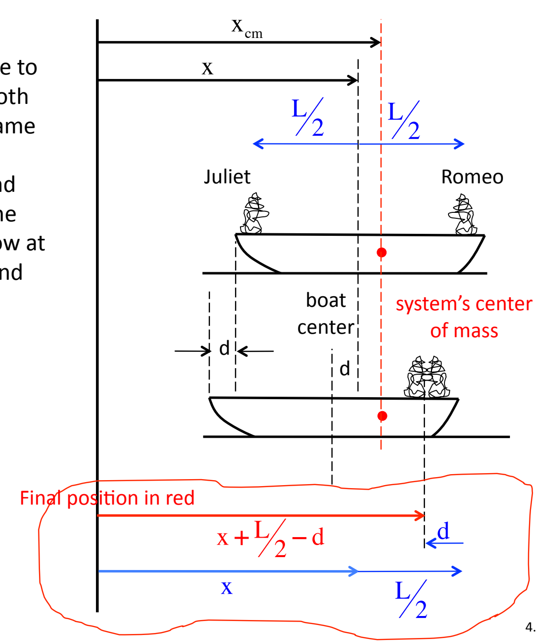
$$\Rightarrow (55 + 77 + 80)x_{cm} = (55)\left(x - 2.7/2\right) + (80)(x) + (77)\left(x + 2.7/2\right)$$

$$\Rightarrow (212)x_{cm} = 55x - 74.3 + (80)(x) + 77x + 104$$

$$\Rightarrow x_{cm} = \frac{55x - 74.3 + (80)(x) + 77x + 104}{212}$$

$$\Rightarrow x_{cm} = x + .14$$

After the boat has moved due to Juliet's change of position, both Juliet and Romeo have the same new position at "x + L/2 – d" (again, look at the boat and the bottom of the sketch). The boat's geometric center is now at the new coordinate "x - d", and the system's *center of mass*, relative to the shore, can be written as:



$$x_{cm} = \frac{\sum m_i x_i}{M}$$

$$\Rightarrow \quad Mx_{cm} = \sum m_i x_i$$

$$\Rightarrow \left(+m_{boat} + \left(m_R + m_J\right)\right)x_{cm} = m_{boat}\left(x - d\right) + \left(m_R + m_J\right)\left(x + L/2 - d\right)$$

$$\Rightarrow \left(55 + 77 + 80\right)x_{cm} = \left(80\right)\left(x - d\right) + \left(77 + 55\right)\left(x + 2.7/2 - d\right)$$

$$\Rightarrow \left(212\right)x_{cm} = \left(80\right)\left(x\right) - 80d + 132x + 178 - 132d$$

$$\Rightarrow x_{cm} = \frac{212x + 178 - 212d}{212}$$

$$\Rightarrow x_{cm} = x + .840 - d$$

The center of mass's location isn't going to change, which means that:

$$x_{cm} = x + .840 - d = x + .14$$

$$\Rightarrow \quad d = .700 \text{ m}$$